Notes: Rational Functions and Asymptotes Name ______________________ Date________

Discontinuities

Holes (removable discontinuity)

- occur when there is a value of x that would make the denominator equal to zero, but we can cancel out the factor

1. Set the canceled factor equal to 0.	2. Solve for x.
3. Plug the answer into the simplified equation to get *y*.	4. (x, y) is the hole

1. $f(x) = \dfrac{x+2}{x^2-4}$

2. $g(x) = \dfrac{x^2-2x-15}{x+3}$

3. $y = \dfrac{4x-20}{x^2-7x+10}$

Vertical Asymptotes (non-removable discontinuity)

- occur when there is a value of x that would make the denominator equal to zero, but we can't get rid of the factor

1. $y = \dfrac{x}{2x-6}$

2. $y = \dfrac{10}{x^2-4}$

3. $y = \dfrac{x+5}{x^2+6x+5}$

To put it simply:

When it comes to the denominator

if it cancels out → hole

if it doesn't cancel out → vertical asymptote

Horizontal Asymptote – occur in 2 out of the 3 cases

- to find a horizontal asymptote, see what happens to the function as x approaches infinity

Conclusion (short cut)

Case 1: → Denominator exponent is larger/higher

Case 2: → Exponents are equal

Case 3: → Numerator exponent is larger/higher

1. $y = \dfrac{1}{2x+5}$

2. $y = \dfrac{5x^2+4x+2}{x^2-7}$

3. $y = \dfrac{x^2-x+2}{x+4}$

Slant Asymptotes

Steps: Use synthetic division to determine the **equation** of the slant asymptote

- occur only when the highest power in the numerator exceeds the highest power in the denominator by exactly one in the simplified function

1. $y = \dfrac{x^2}{x-6}$

2. $y = \dfrac{3x^3-2x+4}{x-3}$

3. $y = \dfrac{3x^2-5x+7}{x-2}$